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The Mechanics of Moving Vertical Fluidized Systems: V. Concurrent Cogravity Flow

J. A. QUINN, LEON LAPIDUS, and J. C. ELGIN

Princeton University, Princeton, New Jersey

An experimental investigation of the concurrent cogravity flow of particulate solids and water in a 1-in. diameter vertical column is reported. Measurements were made of the particle concentration, or holdup, existing in the column as a function of the fluid and particle flow rates for two particle sizes, 0.0184- and 0.00396-in. diameter glass spheres. The experimental results form the basis for a prediction of the generalized characteristics of concurrent cogravity fluidization.

The data for each particle size are correlated in terms of the slip velocity and the holdup. The slip velocity is demonstrated to be the same unique function of the holdup for concurrent cogravity flow and for batch fluidization. Therefore the holdup and the conditions of limiting operation for concurrent cogravity flow can be accurately predicted from the batch fluidization curve.

This publication represents the fifth in a series of investigations of the mechanics of moving fluidized systems. Previous publications (*1, 2, 3, 4*) have discussed a generalized theory of vertical fluidized systems and the experimental validation of this theory as applied to countercurrent and concurrent countergravity flow. As a consequence of this theoretical analysis a relatively new technique for fluid-particle contacting was recognized. The operating characteristics predicted for this type of flow, concurrent cogravity, indicated some rather unique possibilities of contacting, and this investigation was undertaken to study the behavior of such fluid-particle systems.

The theoretical background developed in this program has been detailed by Elgin and Lapidus (*1*), and the salient aspects will be outlined here. Of fundamental importance is the slip velocity defined as the difference between the net fluid and particle velocities. To suitably define this term the convention is adopted that upward flow in a vertical system is positive. It is assumed that the slip velocity is a unique function of the holdup in any fluidized system and thus independent of the relative direction of flow of fluid and particles with respect to each other. The holdup is equal to the volume of particles per unit volume of particles plus fluid.

The relationship between the slip velocity and the holdup can be readily obtained for a fluid-solid system from a batch-fluidization experiment where the net particle velocity is zero and the slip velocity is equal to the average fluid velocity in the bed. The holdup per unit volume is inversely proportional to the expanded bed height; therefore by determining the expanded bed height as a function of the rate of fluid flow into the bed an empirical relation between the slip velocity and the holdup can be obtained for any particular system..

If the slip velocity is a unique function of holdup, the operating characteristics of the various types of vertical fluidized systems can be predicted quantitatively from the results of a single batch fluidization experiment or from one of the generalized correlations which have been published for batch-fluidization data (*5, 6*).

Earlier work in this program has substantiated the predictions concerning the behavior of free countercurrent and free concurrent countergravity flow. Price (*2*) studied three different particle sizes in free countercurrent flow and obtained data in close agreement with predictions from batch-fluidization results. Struve (*3*) investigated concurrent countergravity flow and also substantiated the predicted relation of holdup to slip velocity. Hoffman (*4*) extended the principles to cover the case of mixed-size particles.

At the present time the literature shows only two references to free concurrent cogravity flow. Price (*2*), in the course of his work on countercurrent flow, made one determination of holdup in concurrent cogravity flow. This experiment was performed to demonstrate that higher holdups could be obtained in concurrent cogravity

J. A. Quinn is at the University of Illinois, Urbana, Ill.

flow than in countercurrent flow. Mertes and Rhodes (7) included a few experiments on concurrent cogravity flow. Their reported data are not sufficiently comprehensive to test the validity of the holdup-slip velocity relationship or to generalize the characteristics of this region.

EXPERIMENTAL

Apparatus

Concurrent Cogravity Flow—The essential features of the concurrent cogravity apparatus are shown schematically in Figure 1. The column and the solids feed line were 1-in. Pyrex glass pipe, and all other piping was 1-in. galvanized steel pipe.

Water from the constant head tank was pumped to the top of the column and divided into two streams: one to the column and the other to the by-pass line. The by-pass line was used to maintain a static leg of water in the solids feed line. By varying the flow of water through the by-pass it was possible to maintain the height of water in the solids feed line at a fixed position while varying the flow through the column.

The particles were introduced approximately 1 ft. beneath the by-pass junction, where they rapidly mixed with the water and then flowed through the column and into the solids hopper at the base of the column. The water flowed out of the hopper and then passed through a bank of rotameters and returned to the constant head tank.

That section of the column in which the holdup was measured was subtended at either end by a plug valve with a throat diameter exactly equal to the diameter of the column. With the valves open the column was equivalent to a straight length of 1-in. pipe 9 ft. long. The distance from the point of entry of the particles to the center of the upper plug valve was 11-in. This assured negligible entrance effects. The column extended 18 in. from the lower valve into the solids hopper to minimize any exit effects.

Pressure taps were installed 7 in. below the center of the upper plug valve and 7 in. above the lower valve with a distance of 36½ in. between the two taps. The pressure taps were connected to a *U*-tube differential manometer.

The particles were fed from a hopper above the top of the column, and at the bottom of the hopper the solids flowed through a slide valve equipped with interchangeable orifice plates.

Batch Fluidization—The batch-fluidization experiments were made in a 1-in. Pyrex glass column 7 ft. long. A calming section of 1 ft. of packed coarse sand was placed immediately below the bed support. The column terminated in a *U*-bend which then connected to the constant head tank. The accessory equipment, pump, rotameters, etc., was the same as that used in the concurrent cogravity experiments.

Materials—Two sizes of glass spheres were used in the concurrent cogravity and batch fluidization experiments, grade number 107 and 113. The number 107 spheres passed the U.S. Screen Size number 35 and were retained on number 40; The number 113 spheres passed number 140 and were retained on number 170 screen. The diameter of the beads was taken as the arithmetic average of a sample of 100 beads measured under a microscope with a micrometer eyepiece. The beads were predominantly spherical with less than 5% showing any irregularity in shape.

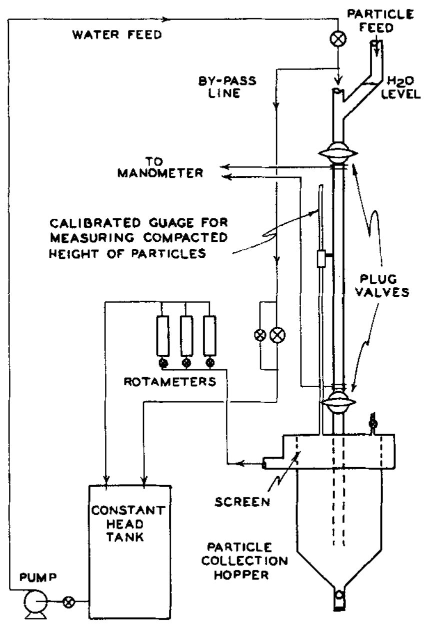

Fig. 1. Schematic of concurrent cogravity experimental setup.

Glass sphere number	Diameter, in.	Density, lb./cu. ft.
107	0.0184	155.0
113	0.00396	153.1

Procedure

Concurrent Cogravity Experiments—A concurrent cogravity run consisted of establishing predetermined fluid and solid rates, maintaining these rates until the system had reached a steady state, recording all pertinent data, closing off the column, and measuring the holdup that existed during the run.

The water flow was adjusted through the column until the water level in the particle feed line was at the correct height and the desired flow through the column was established. The solids hopper was then connected to the particle feed line and the orifice opened. At this point the total flow rate through the column and the liquid level in the side arm were adjusted.

The length of time required to reach a steady state was greatly different for the two particle sizes and was also a function of the holdup in the column. Higher holdups took longer to reach steady state. The attainment of steady state was indicated by a steady pressure drop across the column.

After a steady state had been reached the pressure drop, the rate of solids flow and the water temperature were recorded. Then both the plug valves on the column were closed simultaneously, the pump was shut off, and the solids orifice closed.

After the particles in the column had settled, they were compacted by tapping the column until the solids had reached their maximum consolidation. The height of the compacted solids was then measured.

Batch Fluidization—A representative sample of the beads was dried, weighed, and placed in the batch-fluidization column. The bed was then expanded to the highest voidage attainable, that is to the top of the column, and the water velocity and expanded bed height were recorded at a series of decreasing water velocities until the bed was immobile.

OBSERVATIONS AND EXPERIMENTAL RESULTS

In Figures 2 and 3 the holdup measured as a function of the slip velocity in the batch fluidization and in the concurrent cogravity experiments is depicted for the two particle sizes studied. The batch-fluidization results are represented by the continuous curve (calculated from curvilinear regression of batch data) and the concurrent cogravity results by individual points. The experimentally measured packed bed holdup and the calculated terminal velocity are also plotted for both particle sizes.

These two figures present the crux of the experimental results. The agreement between the concurrent cogravity points and the batch fluidization curve presents a succinct test of the fundamental assumption that for a fluid-solid system the holdup-slip velocity relationship is the same for batch fluidization and for concurrent cogravity flow. Though the concurrent cogravity points show some scatter about the batch fluidization curve, in general the agreement among the data is good.

An equivalent manner of presenting the concurrent cogravity results is shown in Figures 4 and 5. Here the data are shown on a portion of the generalized operating diagram as predicted from the batch-fluidization data. The holdup is given as a function of the superficial fluid velocity at a series of constant particle feed rates. The range of the experimental variables and the agreement of the data with the predicted curves can readily be seen.

For both particle sizes the concurrent cogravity results cover approximately a tenfold variation in the holdup. The lowest values of the holdup, approximately 5%, occur at the minimum particle feed rate and the maximum fluid velocity. At values of the fluid velocity greater than twice the terminal velocity the change in holdup with fluid velocity is small. At

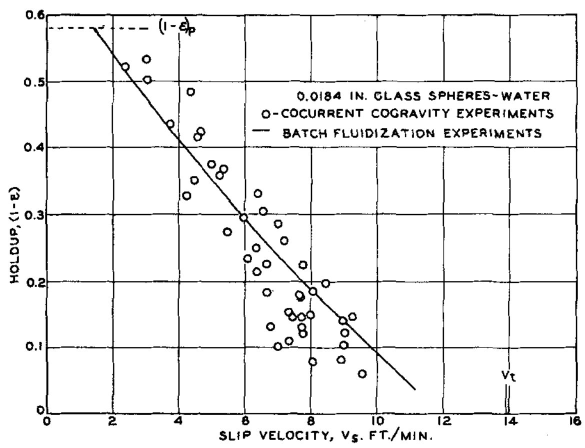

Fig. 2. Experimental holdup vs. slip velocity for 0.0184-in. glass spheres in water.

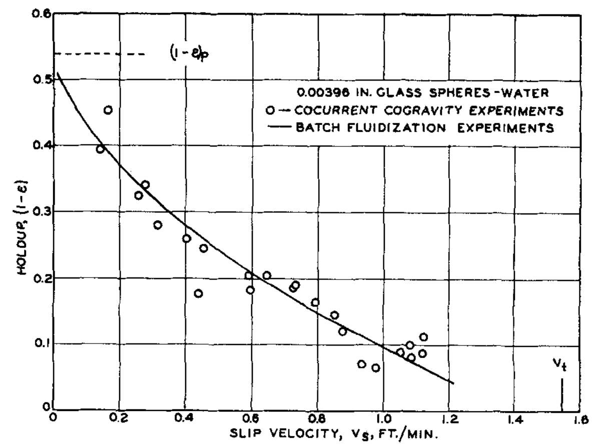

Fig 3. Experimental holdup vs. slip velocity for 0.00396-in. glass spheres in water.

fluid velocities less than the terminal velocity the holdup changes rapidly with fluid velocity and approaches a maximum as the fluid velocity approaches zero.

Experimentally operation at the lowest fluid velocities presented the greatest difficulties. For example in the runs made with the 0.0184-in. beads at a particle rate of 3.91 ft./min. (Figure 4) the predicted holdup changes from 35 to 55% with a corresponding decrease in fluid velocity from 4.0 to 2.4 ft./min. Therefore minute fluctuations in the fluid velocity brought on large variations in the holdup obtained in the column. Actual flooding of the column was achieved at very low fluid velocities. At a particle feed rate of 2.86 ft./min. the maximum holdup obtained was 30.6% at a superficial velocity of 2.81 ft./min. At fluid velocities below 2.8 ft./min. the column could not be operated stably. Below this minimum value the particle feed to the column fluctuated with particles backing up into the particle feed leg. Also the holdup in the column was not uniform.

The operating characteristics described above fixed the upper limit on the holdup that could be obtained at any particle feed rate. For the runs made with the 0.0184-in. beads the maximum holdup obtained at each particle feed rate (Figure 4) represents the highest holdup attained with stable operation.

The highest holdup obtained with the 0.0184-in. beads was 53% at a particle feed rate of 4.61 ft./min. The packed-bed holdup was measured as 58% in the batch-fluidization experiments. However this value represents the holdup at *minimum consolidation* of the particles corresponding to the loosest stable packing of the particles and is subject to considerable variation (*8*). Struve (*3*) reports a value of 52.6% for the packed-bed holdup of particles having the same diameter as the 0.0184-in. beads used in this work. Therefore the maximum holdup of 53% obtained in the concurrent cogravity experiments closely approximates the packed-bed holdup and may be taken as confirmation of the fact that in concurrent cogravity flow the particles can be passed through the column at the packed-bed holdup.

In all of the runs with the 0.0184-in. beads shown in Figure 4 the column operation was quite stable. Over the complete range of particle and fluid rates covered the holdup was uniform over the whole column and no gross circulation patterns were detected in the particle flow path. The only anomalies detected in the column operation occurred at very low fluid velocities such as described above. The results of the concurrent cogravity experiments made with the 0.00396-in. beads, Figure 5, are analogous to those obtained with the larger particles, the 0.0184-in. beads. However because of the smaller terminal velocity, 1.55 ft./min. for the 0.00396-in. beads vs. 13.9 ft./min. for the 0.0184-in. beads, correspondingly longer operating times were required to attain steady state conditions. These longer operating times placed an upper limit on the maximum holdup that could be obtained at the lowest fluid velocities.

The agreement between the batch data and the concurrent cogravity results for the 0.00396-in. beads is better than for the 0.0184-in. beads because extraneous factors such as a wall effect are smaller with the smaller beads. Also the longer operating times in the case of the 0.00396-in. beads resulted in a closer approach to a true steady state than in the runs with the 0.0184-in. beads. In general for a given fluid-solid system a smaller particle size gives a better quality of fluidization.

The scatter of the points in Figures 2 and 3 arises partially from the arithmetic operations performed in calcu-

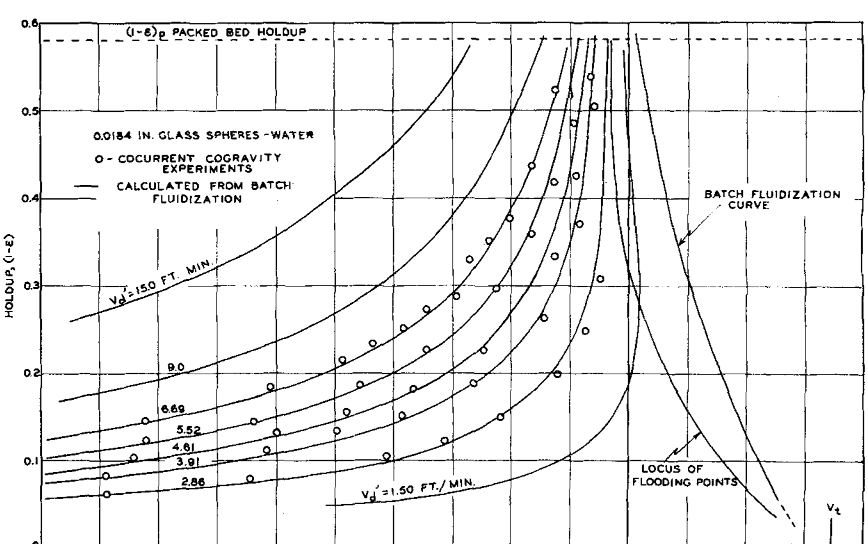

Fig. 4. Experimental holdup vs. superficial fluid velocity for 0.0184-in. glass spheres in water.

lating the slip velocity from the equation

$$V_s = \frac{V_d'}{1-\epsilon} - \frac{V_f'}{\epsilon}$$

The quotients $V_d'/1-\epsilon$ and V_f'/ϵ represent the ratios of two measured quantities, each of which is subject to error. The difference of these two quantities then represents the slip velocity, and the greatest loss of accuracy can result in this differencing operation. Probable errors of approximately 3% in the measured quantities may result in a variation of 10% in the calculated slip velocity.

CONCURRENT COGRAVITY CONTACTING

The experimental results of this investigation clearly demonstrate that higher holdups can be attained in concurrent cogravity flow than in countercurrent flow and that these higher holdups can be attained at much lower fluid velocities than in concurrent countergravity flow. Moreover the results indicate that particles can be passed through a concurrent cogravity column at approximately the concentration existing in the packed or quiescent bed. Although the behavior of concurrent cogravity flow has been demonstrated here for only two sizes of rigid particles and one fluid, these results form a basis for a consideration of the characteristics of concurrent cogravity contacting.

The limits of operation for concurrent cogravity fluidization can be seen in Figures 4 and 5. Although these diagrams pertain to two definite systems, their general outline is analogous to that which would be exhibited by a wide range of fluid-particle systems. Stable operation of a system in free concurrent cogravity flow is represented by the entire region extending from zero holdup to the packed bed holdup for all fluid velocities greater than zero with the exception of the small area enclosed by the locus of flooding points. The locus of flooding points represents the maximum holdup which can be obtained at any fluid velocity. Primarily flooding is a phenomenon associated with countercurrent operations. However the locus of flooding points forms a continuous curve which extends into the concurrent cogravity region and limits the holdup attainable at very low fluid and particle rates. That area of the concurrent cogravity region enclosed by the vertical line of zero fluid velocity, the horizontal line representing the packed-bed holdup, and the locus of flooding points have been shown to represent restrained concurrent cogravity flow (8). The operation of a free system will terminate at the flooding curve, and to attain a holdup greater than that at flooding, at a fixed particle rate, a restriction must be placed in the flow path; this restriction could be a screen placed at the exit of the column, and the particle flow rate would be controlled externally at the particle exit.

The principal advantage of concurrent cogravity flow over countercurrent and transport flow is that it has a considerably broader range of operation. For a system in countercurrent flow the maximum range of fluid velocity is from zero to the terminal velocity of a single particle. As the particle flow rate is increased, the range of fluid velocities diminishes. The maximum holdup attainable in countercurrent flow occurs at a zero fluid velocity and for most systems does not exceed approximately 30%. Concurrent countergravity flow is limited in that it can be realized only at fluid velocities greater than the terminal velocity of a single particle.

In concurrent cogravity contacting very high holdups can be obtained at very low particle and fluid rates. In certain applications this type of contacting might have distinct advantages. Concurrent cogravity flow offers a possibility of continuously contacting two streams at very low flow rates and achieving the maximum contact area between the two streams.

This type of flow is limited in transfer processes as are all concurrent operations. The over-all driving force for heat or mass transfer is generally smaller in concurrent than countercurrent flow. However a smaller temperature or concentration gradient might be more than compensated for by the greater interfacial area for transfer that is possible in this type of flow.

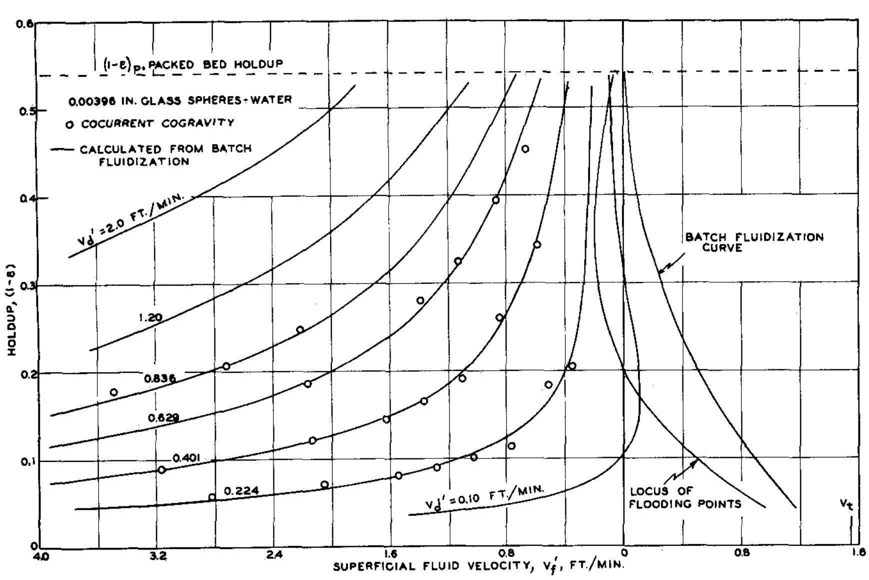

Fig. 5. Experimental holdup vs. superficial fluid velocity for 0.00396-in. glass spheres in water.

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NOTATION

- V_d' = superficial particle velocity based on total column cross-sectional area, ft./min.
- V_f' = superficial fluid velocity based on total column cross-sectional area, ft./min
- V_s = average slip velocity, ft./min.
- V_t = terminal velocity of single particle, ft./min.
- ϵ = fraction, or percent, voidage
- $(1-\epsilon)_p$ = holdup of packed bed

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